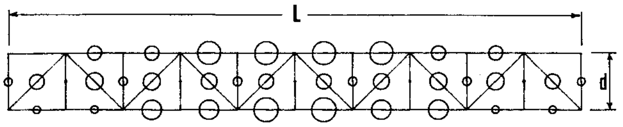
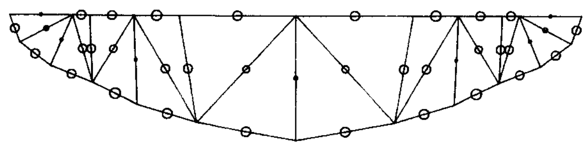
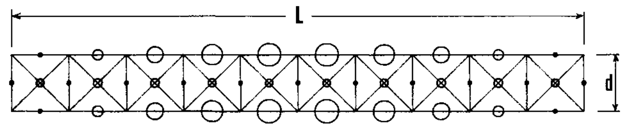


a) **Initial configuration ($d/L = 0.1$)**

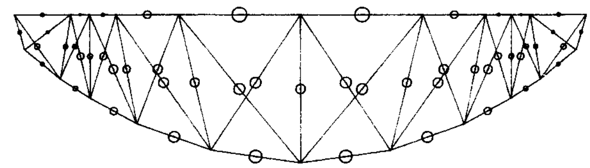


b) **Optimal configuration**

Fig. 1 Sensitivities of two-dimensional truss for mean square error (statically determinate truss, $N_s = 10$).



a) **Initial configuration ($d/L = 0.1$)**



b) **Optimal configuration**

Fig. 2 Sensitivities of two-dimensional truss for mean square error (statically indeterminate truss, $N_s = 10$).

Table 1 Surface error ratio of optimal configuration

Type of truss mast	Surface error ratio ($\epsilon^2_{\rm opt}/\epsilon^2_0$) (Nondimensional optimal error $\epsilon^2_{\rm opt}/\sigma^2_\epsilon L^2$)			
	Number of subdivisions, N_s			
	4	6	8	10
Statically determinate truss	0.233 (0.0900)	0.330 (0.0639)	0.478 (0.0610)	0.548 (0.0532)
Statically indeterminate truss	0.206 (0.0527)	0.297 (0.0428)	0.376 (0.0375)	0.456 (0.0349)

rations. In these figures, the area of the circle displayed on each element indicates the sensitivity levels. By comparing the statically determinate truss of the initial configuration with the statically indeterminate one, the diagonal members of the determinate truss are more sensitive for the mean square errors than those of the indeterminate one. The figures also indicate that the locations of the effective members are symmetric, and the central longitudinal elements are most effective. Thus, as for the effects of the diagonal members on the mean square errors, the statically determinate truss is more sensitive. This is why the mean square errors of the statically determinate truss are greater than those of the statically indeterminate one, although there are fewer members. These figures show that the indeterminate truss can decrease the effects of diagonal members on the mean square errors, but for the contribution of the longitudinal member on the errors, the difference between the indeterminate truss and the determinate one is very small for the initial configurations. For the optimal configuration, it is shown that the effects of the longitudinal member on the square errors can be reduced, and therefore, the optimal shape is less sensitive in changing the element lengths.

Table 1 shows ratios of the nondimensional mean square errors of upper surface displacement of the initial and the optimal truss masts vs the number of subdivisions of the initial truss mast. For the initial configuration, the errors (ϵ_0) have been calculated under the same truss depth ($d/L = 0.1$). These results indicate that the optimal configuration is more effective for reducing the structural error in the case of fewer subdivisions, although the mean square error of the optimal configuration increases as the number of subdivisions decreases. For the efficiency of optimization, the statically indeterminate truss is better than the statically determinate one.

Concluding Remarks

The optimal shape design for structural accuracy of space structures based on a sensitivity analysis has been presented. The results of the two-dimensional truss masts have been demonstrated to predict the effective members contributing to the errors, and the characteristics of the optimal configuration have been investigated.

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Displacement Approximations for Optimization of Beams Defined in Nonprincipal Coordinate Systems

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Introduction

APPROXIMATION concepts have been instrumental to many of the advances in structural optimization.[1,2] Early structural optimization methods used finite element analysis to evaluate the structural response throughout the optimization procedure. This approach required large numbers of analyses and was costly, particularly for large problems. By approximating the structural response, it became possible to eliminate many of these costly finite element analyses. The earliest approximations were in the form of first-order Taylor series, expanding the responses directly in terms of the design variables or their reciprocals. In truss structures, where the

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member areas A are the design variables, Taylor series expansions for the displacements in terms of $1/A$ are exact for statically determinate structures and preferred for statically indeterminate structures. Similarly, for beam structures, displacement approximations are usually formed in terms of the reciprocal section properties.[3,4]

For beam structures, we may approximate the structural responses in terms of either the section properties in the principal axis system or some other nonprincipal system. In the past, optimization was done with simple symmetric sections and it was natural to form the approximations as a function of the reciprocal principal section properties. As we have moved to more sophisticated design elements with general cross sections (Fig. 1), it has been more convenient to define the elements in a local coordinate system which is typically not the principal axis system. In these cases, it would be natural to construct the approximations for the structural response quantities in terms of the reciprocals of the nonprincipal section properties. In this Note, we show that the choice of section properties used in the approximation is critical. Approximations based on the reciprocals of the principal section properties are exact for statically determinate beam structures when the orientation of the principal axes remains constant. This is not the case when the approximations are based on the reciprocals of the nonprincipal section properties. Our computational results show that the errors from expansion in the nonprincipal section properties can be significant and can prevent convergence of the optimal design process even when small move limits are employed.

Analytical Results

Consider the beam shown in Fig. 1. The displacements u_y and u_z are given by Ref. 5:

$$u_z = (FL^3/3E)[I_z/(I_yI_z - I_{yz}^2)] \tag{1a}$$

$$u_y = (FL^3/3E)[I_{yz}/(I_yI_z - I_{yz}^2)] \tag{1b}$$

If $I_{yz} = 0$ then $u_y = 0$ and $u_z = FL^3/3EI_y$ is linear in the reciprocal section property $1/I_y$. Thus, the first-order Taylor series approximation about I_{y0} with $u_{z0} = u_z(I_{y0})$

$$\tilde{u}_z = u_{z0} + \frac{\partial u_z}{\partial(1/I_y)} \Delta(1/I_y) \tag{2}$$

is exact since $\partial u_z/\partial(1/I_y) = FL^3/3E$ is a constant.

When $I_{yz} \neq 0$, these simplifications can no longer be made and the resulting first-order Taylor series

$$\tilde{u}_z = u_{z0} + \frac{\partial u_z}{\partial(1/I_y)} \Delta(1/I_y) + \frac{\partial u_z}{\partial(1/I_z)} \Delta(1/I_z) + \frac{\partial u_z}{\partial(1/I_{yz})} \Delta(1/I_{yz}) \tag{3}$$

is insufficient to exactly represent u_z as I_y, I_z, and I_{yz} vary.

Next, we consider u_z as a function of the principal moments of inertia I_1, I_2, and the angle θ which defines the orientation of the principal axes as shown in Fig. 2. The following equations relate the moments of inertia in system (1, 2) to those in the (y, z) system:

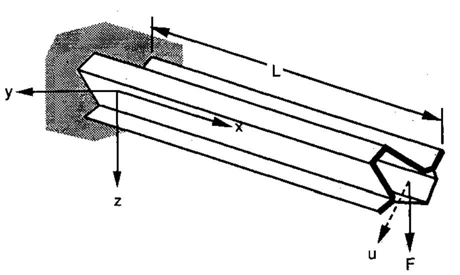

Fig. 1 Thin-walled cantilevered beam with general cross section.

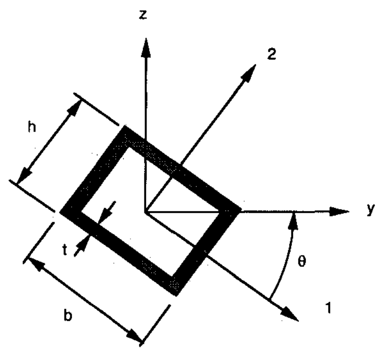

Fig. 2 Relationship between principal and nonprincipal axes.

$$I_y = I_1 \cos^2\theta + I_2 \sin^2\theta - 2I_{12} \sin\theta \cos\theta \tag{4a}$$

$$I_z = I_1 \sin^2\theta + I_2 \cos^2\theta + 2I_{12} \sin\theta \cos\theta \tag{4b}$$

$$I_{yz} = (I_1 - I_2) \sin\theta \cos\theta + I_{12}(\cos^2\theta - \sin^2\theta) \tag{4c}$$

Assuming (1, 2) is the principal system, we have $I_{12} = 0$. Using these relationships, and noting that $I_yI_z - I_{yz}^2 = I_1I_2$ we may write Eq. (1) as

$$u_z = \frac{FL^3}{3E}\left(\frac{\sin^2\theta}{I_2} + \frac{\cos^2\theta}{I_1}\right) \tag{5a}$$

$$u_y = \frac{-FL^3}{3E}(1/I_2 - 1/I_1) \sin\theta \cos\theta \tag{5b}$$

which is clearly linear in $1/I_1$ and $1/I_2$. This suggests that the alternative Taylor series expansion in terms of the reciprocal principal section properties and θ

$$\hat{u}_z = u_{z0} + \frac{\partial u_z}{\partial(1/I_1)} \Delta(1/I_1) + \frac{\partial u_z}{\partial(1/I_2)} \Delta(1/I_2) + \frac{\partial u_z}{\partial\theta} \Delta\theta \tag{6}$$

will be exact for constant θ.

Specific steps may be taken to insure that the displacement derivatives obtained from the sensitivity analysis (e.g., Ref. 6) will be computed with respect to the principal moments of inertia,[7] allowing us to form the expansion of Eq. (6) directly. However, in general, it is preferable to compute the displacement derivatives with respect to the principal section properties from the derivatives with respect to the nonprincipal section properties by using the following chain rule formulae:

$$\frac{\partial u_z}{\partial(1/I_1)} = \left[\frac{\partial u_z}{\partial I_y}\frac{\partial I_y}{\partial I_1} + \frac{\partial u_z}{\partial I_z}\frac{\partial I_z}{\partial I_1} + \frac{\partial u_z}{\partial I_{yz}}\frac{\partial I_{yz}}{\partial I_1}\right]\frac{\partial I_1}{\partial(1/I_1)} \tag{7a}$$

$$\frac{\partial u_z}{\partial(1/I_2)} = \left[\frac{\partial u_z}{\partial I_y}\frac{\partial I_y}{\partial I_2} + \frac{\partial u_z}{\partial I_z}\frac{\partial I_z}{\partial I_2} + \frac{\partial u_z}{\partial I_{yz}}\frac{\partial I_{yz}}{\partial I_2}\right]\frac{\partial I_2}{\partial(1/I_2)} \tag{7b}$$

$$\frac{\partial u_z}{\partial\theta} = \frac{\partial u_z}{\partial I_y}\frac{\partial I_y}{\partial\theta} + \frac{\partial u_z}{\partial I_z}\frac{\partial I_z}{\partial\theta} + \frac{\partial u_z}{\partial I_{yz}}\frac{\partial I_{yz}}{\partial\theta} \tag{7c}$$

where the partial derivatives of the displacements with respect to the nonprincipal moments of inertia are obtained directly from the sensitivity analysis. The partial derivatives of the nonprincipal section properties with respect to the principal section properties and θ are obtained by direct differentiation of Eq. (4) with $I_{12} = 0$.

Computational Results

To compare the accuracy of the approximations given by Eqs. (3) and (6), consider the design of the simple cantilevered beam shown in Fig. 1 with the rectangular cross section shown in Fig. 2. We took $L = 120$ in. (304.8 cm), $F = 1000$ lbf (4448 N), $E = 29 \times 10^6$ psi (20×10^6N/cm^2), with $\theta = -30$ deg and $t = 0.25$ in. (0.635 cm). The displacement in the direction of F was required to be less than 0.24 in. (0.61 cm). The optimal design, generated using the program described in Ref. 8, has a mass of 267 lb (121.11 kg) with $b^* = 6.84$ in. (17.37 cm) and $h^* = 9.09$ in. (23.09 cm). By aligning the beam element orientation vector with the principal axes, we were able to approximate the displacement in terms of the reciprocal principal moments of inertia as in Eq. (6). With sufficiently large move limits, convergence to the optimal design was achieved in one step (requiring only a single evaluation of the tip displacement and its derivatives) from any starting point. This confirmed that the displacement approximation was exact when formed using the reciprocal principal section properties. Next, defining the beam element orientation vector along the global Y axis, we approximated the displacement in terms of the reciprocals of the nonprincipal moments of inertia as in Eq. (3). Starting at $b = 5.5$ in. (13.97 cm) and $h = 8.5$ in. (21.59 cm), which is quite close to the optimal design, convergence was prevented by errors in the approximation $\tilde{u}_z$ of u_z, in spite of the rather small move limits of 10%.[7]

We compared the approximation $\tilde{u}_z$ to the exact displacement u_z by computing $\tilde{u}_z/u_z$ analytically for various values of h/h_0 and b/b_0 using the rotated rectangular beam discussed above. Taking $b_0 = 5.5$ in. (13.97 cm) and $h_0 = 8.5$ in. (21.59 cm), we computed $\tilde{u}_z/u_z$ for b/b_0 and h/h_0 between 0.75 and 1.25. The largest approximation errors (nearly 800%) occurred when $b \simeq h$ and thus $I_1 \simeq I_2$ and $I_{yz} \simeq 0$. At these points $\Delta(1/I_{yz})$ is nearly singular causing large errors in $\tilde{u}_z$. However, significant errors occur even when $b \neq h$. For example, in the range $0.75 \leq b/b_0 \leq 1.1$ and $0.85 \leq h/h_0 \leq 1.25$, Fig. 3 shows the largest error $\tilde{u}_z/u_z = 0.83$ for $b/b_0 = 1.1$ and $h/h_0 = 0.85$, which corresponds to $h = 7.225$ in. (18.35 cm) and $b = 6.05$ in. (15.37 cm).

We also evaluated the approximation

$$\bar{u}_z = u_{z0} + \frac{\partial u_z}{\partial(1/I_y)}\,\Delta(1/I_y) + \frac{\partial u_z}{\partial(1/I_z)}\,\Delta(1/I_z) + \frac{\partial u_z}{\partial I_{yz}}\,\Delta I_{yz} \quad (8)$$

which eliminates the singularity which occurs in Eq. (3) when $I_{yz} = 0$. The values of $\bar{u}_z/u_z$ are shown in Fig. 4 for b/b_0, h/h_0 between 0.75 and 1.25 with $b_0 = 5.5$ in. (13.87 cm) and $h_0 = 8.5$ in. (21.59 cm). Although this form of the approximation eliminates the large errors in u_z when I_1 is near I_2, it is less

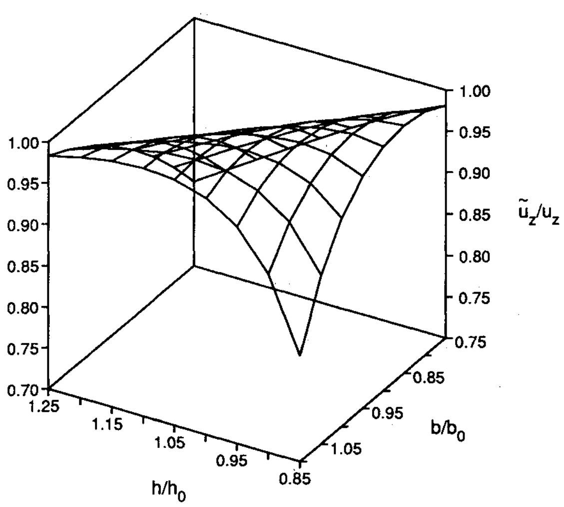

Fig. 3 Errors in $\tilde{u}_z$ as a function of b and h. $b_0 = 5.5$ in. (13.87 cm), $h_0 = 8.5$ in. (21.59 cm).

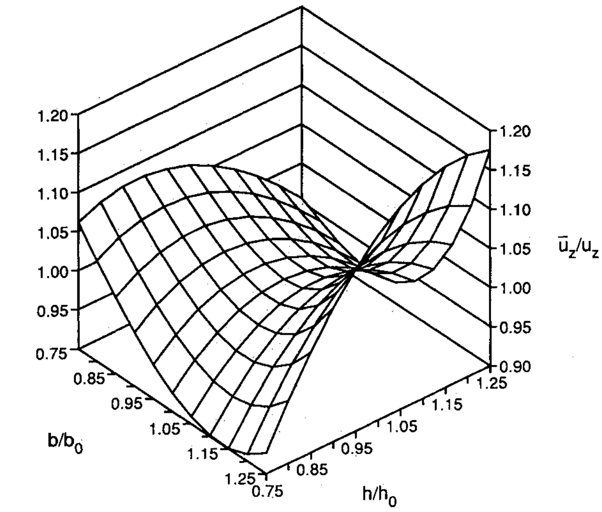

Fig. 4 Errors in $\bar{u}_z$ as a function of b and h. $b_0 = 5.5$ in. (13.87 cm), $h_0 = 8.5$ in. (21.59 cm).

accurate than the expansion in terms of the principal section properties [Eq. (6)].

The accuracy of the displacement approximations was also studied for an idealized automobile frame structure.[7] As in the case of the cantilevered beam, large errors (as large as 724% for 25% move limits and 897% for 50% move limits) occurred when the displacement constraints were approximated in terms of the reciprocal nonprincipal section properties. In comparison, the largest error was 17% (for 50% move limits) when the displacement constraints were approximated using the reciprocal principal section properties.

Discussion

For structural beam members with cross-sectional properties described in a nonprincipal coordinate system, we have shown that a displacement approximation using the reciprocal nonprincipal section properties [Eq. (3)] can give very poor results. This may occur in simple statically determinate frame structures as well as complex structures and is due, in part, to the theoretical behavior of the approximation. When θ, the principal axis orientation, remains constant the approximation written in terms of the reciprocal principal section properties will be exact for statically determinate structures while the approximation with nonprincipal properties is not. In addition, numerical difficulties which occur as I_{yz} approaches 0 may cause very large errors in the approximate displacements when the nonprincipal properties are used.

Although we have restricted our analysis to those cases where the orientation of the principal axes remains fixed, this is often satisfied in practice. For example, sections which remain symmetric during design will satisfy this restriction, as will arbitrary sections whose design variables allow only scaling of the section. Our new method [Eq. (6)] constructs the approximation as a function of the reciprocal principal section properties and provides much better displacement approximations, even when large move limits are employed for a realistic statically indeterminate frame structure. Further study will be required to determine the best approximation to be used when the principal axis orientation does not remain fixed during optimization.

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Errata

Thin Film Modeling of Delamination Buckling in Pressure Loaded Laminated Cylindrical Shells

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DURING printing, page 2123 of this article was inadvertently exchanged with page 2133. We regret this error.

Stacking Sequence Optimization of Simply Supported Laminates with Stability and Strain Constraints

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DURING printing, page 2133 of this article was inadvertently exchanged with page 2123. We regret this error.